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A Case Study on Six Sigma-Based Bayesian Control Charts under Gamma Prior Distribution

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Abstract

Statistical process control, or SPC, is a proactive strategy that uses statistical methodology to monitor a process control.In contrast to acceptance sampling plans, which are typically employed after nonconformity products have already occurred in the process, SPC is used to identify process abnormalities once they spiral out of control and after appropriate preventive measures have been made to manage the production process. In particular, it is the use of statistical techniques to gather data, chart the data, and then track the variability of a specific process of interest over a period of time in relation to the lower and upper control limits, which are typically set at three standard deviations above and below the process target line.The Bayesian approach is a novel and intriguing method in industrial statistical quality control (SQC) that relies on the subjective likelihood of past information about the process. In this research paper, the construction of six sigma-based control chart is developed by using Gamma prior distribution along with Bayesian approach.

1. INTRODUCTION

Bayesian approach incorporates many forms of uncertainty into the model and expresses them.The Bayesian context gives an overview of the steps taken before and after data observation, as well as the final product. In reaction to fresh data collection, it also enables the dynamic updating of estimated parameters and control chart elements. Interesting system parameters are defined by Bayesian models as variables that exhibit a behavior inside an uncertain probability distribution. The structure of observable and unobservable variables, parameters, and their connections are shown by the Bayesian model.This structure aims to solve the production system's parameters while maintaining greater flexibility. The Bayes approach, which is used in process monitoring and evaluation of process control, differs from the traditional approach in that it addresses uncertainty about the relevant parameter by combining sample data with previous knowledge. Ulrich (2002, 2007) verified the control charts variability and its aim using the Bayes framework. Ulrich (2010) recommended the use of a constructed control chart in situations when the process's aim and its variance are uncertain in a normal distribution. In order to update the process, mean and display the control limits, Aamir proposed the posterior control limits chart in 2015. He did this by exploiting the context of the informative and non-informative priors. Aunali and Venkatesan (2017) recently examined the comparison of the classical control charts and the Bayesian technique for detecting tiny shifts.

2. BAYESIAN APPROACH FOR GAMMA PRIOR DISTRIBUTION

Estimating a summary of prior distributions and current sample data is based on the Bayesian method to statistical analysis and modeling. The statistical process is identified by computing the average efficiency of all conceivable data, as defined by statistical theory.This is where the Bayesian process contradicts itself because it focuses more on the behavior of the process in a particular circumstance. Furthermore, in contrast to frequentist methods, Bayesian techniques formally incorporate data from sources other than statistical surveys. The probability distribution of the unknown parameter in a statistical model is described by this data, which is historical experiences. In cases where statistical results and decisions are dubious, Bayesian techniques provide an excellent model. Statistical approaches are commonly employed to address numerous issues encountered by conventional statistical methods and to enhance their utility. Statistical inferences for the precise interest in the Bayesian framework are known as obvious value-induced unpredictability adjustments.

$$\textit{Posterior} \propto (\textit{Likelihood} \times \textit{Prior}\ )$$

Later information about the sample size is referred to as "Posterior" in this context. "Likelihood" is a prototype observation, and "prior" refers to being aware of the interest's quantity in the probability distribution. As a result, three random values of X and the scale and shape parameters, α and β, respectively, are assigned with the Bayes theorem as follows:

$$p(\alpha,\beta|X)\ \propto L(X)\times p(X|\alpha,\beta)$$

The posterior distribution, $p(\alpha,\beta|X)$, is formed by combining the prior distribution and the likelihood function.

In the Bayesian technique, fixing the prior distribution is a very challenging issue that depends on the problem and knowledge, i.e., informative vs non-informative. The Gamma prior distribution is taken into consideration in this article, which considers informative priors.

This can be a good way to illustrate a common relation.

$$E[\alpha,\beta]=E[E(\alpha,\beta|X)] \qquad (1)$$

$$E[\alpha,\beta]=E[Var(\alpha,\beta|X)]+Var[E(\alpha,\beta|X)] \qquad (2)$$

The first equation demonstrates that the expected of all potential posterior means is the mean's prior (using every dataset that might be used for process control). The second illustrates how the variation in the posterior means is on average.

Gamma distribution, which is used to create the control chart, is one way to quantify the previous data. It is provided by

$$P(X|\alpha,\beta)=\frac{\alpha^{\beta}}{\Gamma(\beta)}e^{-\alpha x}\,x^{\beta-1}; \qquad x\geq 0;\alpha,\beta>0$$

Given a random sample of $X_{1,}X_2,\dots,X_n$, the likelihood function may be expressed as

$$L\propto\left[{\alpha^{\beta}}/{\Gamma(\beta)}\right]^{n}e^{-\alpha\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}\left(x_i^{\beta-1}\right)$$

After using certain simplifying assumptions, the joint posterior density function can be written as follows using Baye's theorem:

$$P(\alpha,\beta|X)\propto\left[\left\{\frac{\alpha^{\beta}}{\Gamma(\beta)}\right\}^{\mathrm{n}+1}e^{-\alpha\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}x_i^{\beta-1}\right]\times\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}e^{-\alpha x}x^{\beta-1}\right]$$

$$=\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n+1}e^{-\alpha\sum_{i=1}^{n+1}x_i}\prod_{i=1}^{n+1}x_i^{\beta-1} \qquad (3)$$

After making the necessary simplifications, the posterior marginal r^{th} moments $E(\propto,\beta|X^r)$ can be found as follows

$$E(\alpha,\beta|X^r)=\int_0^{\infty}x^r\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n+1}e^{-\alpha\sum_{i=1}^{n+1}x_i}\prod_{i=1}^{n+1}x_i^{\beta-1}dx$$

$$= \left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n+1} \frac{\Gamma(\beta + r + 1)}{\alpha^{\beta+r-1}}$$

The first posterior and second posterior moments are then obtained by further simplifying the expressions and replacing r=1 and r=2, respectively. These are provided by

$$E(\alpha, \beta|X) = \left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n+1} \frac{\Gamma(\beta + 2)}{\alpha^{\beta}} = \beta(\beta + 1)\left[\frac{\alpha \quad \beta}{\Gamma(\beta)}\right]^{n} \tag{4}$$

$$E(\alpha, \beta|X^2) = \left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n+1} \frac{\Gamma(\beta + 3)}{\alpha^{\beta+1}} = \beta(\beta + 1)(\beta + 2)\frac{\alpha^{n\beta-1}}{[\Gamma(\beta)]^{n}} \tag{5}$$

The posterior variance, which is provided by equations (4) and (5) after simplifications, can be obtained by

$$V(a, b|X) = \beta(\beta + 1)(\beta + 2)\frac{\alpha^{n\beta-1}}{[\Gamma(\beta)]^{\mathrm{n}}} - \beta^2(\beta + 1)^2\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{2n} \tag{6}$$

$$= \frac{\beta(\beta + 1)}{[\Gamma(\beta)]^{2\mathrm{n}}}\left\{(\beta + 2)\alpha^{n\beta-1}[\Gamma(\beta)]^{n} - \beta(\beta + 1)\alpha^{2n\beta}\right\}$$

3. CONTROL LIMITS FOR GAMMA PRIOR DISTRIBUTION

Decisions about the state of process control are made using a Bayesian control chart, which shows the probability of two out of control levels.Although it requires considerably more process structure information than is often provided by control charts, this knowledge can have significant advantages.If the sample size is insufficient, the posterior variance will be less than the prior variance, indicating a different distribution based on the data and prior of the distribution.

As a result, equations (4) and (6) can be used to determine the process control's limits.

$$UCL = \beta(\beta + 1)\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n} + 3\sqrt{\frac{\beta(\beta + 1)}{[\Gamma(\beta)]^{2n}}\{(\beta + 2)\alpha^{n\beta-1}[\Gamma(\beta)]^{n} - \beta(\beta + 1)\alpha^{2n\beta}\}}$$

$$CL = \beta(\beta + 1)\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n}$$

$$UCL = \beta(\beta + 1)\left[\frac{\alpha^{\beta}}{\Gamma(\beta)}\right]^{n} - 3\sqrt{\frac{\beta(\beta + 1)}{[\Gamma(\beta)]^{2n}}\{(\beta + 2)\alpha^{n\beta-1}[\Gamma(\beta)]^{n} - \beta(\beta + 1)\alpha^{2n\beta}\}}$$

4. Six sigma

The term "Six Sigma" refers to a statistical measure of how far a process deviates from perfection. A process that operates at six sigma has a failure rate of only 0.00034%, which means it produces virtually no defects. Six Sigma was developed by Motorola in the 1980s, and it has since been adopted by many other companies around the world, including General Electric, Toyota, and Amazon. It is used in industries such as manufacturing, healthcare, finance, and service industries to improve customer satisfaction, reduce costs, and increase profits. Radhakrishnan and Balamurugan (2010) amassed six sigma based totallyExponentially Weighted Moving Average Control (EWMA) frame. The short designs began by using W.A. Shewhart (1931) changed into set up totally three sigma, if all else fails, control limits.

5. Conclusion

The Bayes technique incorporates previous information about unknown parameters into the procedure by allocating the parameters to the Gamma prior distribution. To obtain the posterior distribution, the prior information was combined with the likelihood function to create control charts based on six sigma using the Bayesian technique. The posterior distribution yields a future distribution of the standardized mean. It is recommended that the sample size be kept to a minimum; otherwise, control limit deviations will be significantly closer to zero. If the quality engineer has a large number of observations and a large sample size, it is recommended to utilize the generalized the parameters.

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