South Asian Journal of Engineering and Technology Research Article

Open Access Full Text Article 146(2024) 83- 86 DOI: 10.26524/sajet.2024.14.28

Construction of Six-Sigma Based Control Charts Under Exponentiated Mukherjee - Islam Distribution for Statistical Process Control

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Article info
Received 7th August 2024 Received in revised form 8 September 2024 Accepted 6 October 2024

Keywords
Control limits, Exponentiated Mukherjee-Islam Distribution (EMID), Length-biased distributions, Statistical Process Control.

https://sajet.in/index.php/journal/article/view/298

Abstract
In the domain of process monitoring, statistical process control, or SPC, is frequently employed when the quality of interest is normally distributed. In actuality, the variable of interest may follow any non-normal distribution, such an exponential, gamma, or other distribution. It is not necessarily true that the variable of interest follows the normal distribution. It might not be feasible to employ control charts made for a normal distribution in this case, which could lead to an increase in the percentage of non-conforming goods. This research article employs Length Biased distribution strategies to evaluate the performance of the production process using the EMID through the six-sigma based control charts. This research paper's primary goal is to study and monitor the production system by introducing a six-sigma based control chart utilizing the New Generalization of EMID.

1. INTRODUCTION

The continuous improvement technique known as statistical process control, or SPC, is widely used in contemporary manufacturing and service companies. The utilization of control charts and frequency distributions of development and quality data is the main emphasis of this methodology. One popular and well-respected control chart is the Shewhart control chart. Control chart usage was initially suggested by W. A. Shewhart. Essential tools for quality assurance are control charts. Control charts are used to monitor and manage existing processes. They can be used to identify and address issues as they arise, predict the estimated range of outcomes from a process, assess process stability, look into patterns of process variation resulting from special causes, non-routine events, or common causes incorporated into the process, and decide whether to prevent specific issues or alter the process fundamentally as part of a quality improvement project. Control charts aid in the creation of better products. When a control chart indicates that the process is under control, that is, no action should be taken. Control charts are meant to be utilized to take timely action on the process. On the other hand, appropriate action should be taken to regain control if the control chart shows that the advancement has become uncontrollable. Because of this, a control chart indicates which statements should receive remedial action and when. The progression target means or variation is monitored using the two control limits, which are referred to as the upper and lower control limits (UCL and LCL). These limitations are very helpful in reducing defective products and raising company profits. Control charts help an industry maintain its good reputation in the marketplace by preserving quality.

Shewhart control charts are widely used to monitor activities when the quality of interest has a normal distribution. The variable of interest in a run-through may or may not follow a non-normal

distribution instead of the normal distribution. Numerous academics have produced various kinds of control charts. The Bayesian Approach in Control Charts Techniques was also covered by Amin and Venkatesan (2019) along with the latest advancements in control chart approaches. The t-chart is suggested as a control chart by Santiago and Smith (2013) when the intervals between events have an exponential distribution. They used exponentially distributed data and Nelson's variable transformation (1994) to approach normal data. The length biased weighted Erlang distribution was identified and its characteristics and applications were explained. Rather and Subramanian (2019) investigated the length biased Erlang-truncated exponential distribution using life time data. This situation may prevent control charts designed for a normal distribution from being applicable, which would increase the amount of nonconforming goods. Additionally, when data is collected in subgroups, the normal distribution is employed so that control charts can be made using the central limit theorem. In practice, gathering data in groups isn't always feasible. Because of this, classical distributions often do not give enough correction for actual data. The normal distribution, for instance, is not a good option if the data is asymmetric. To create novel distributions, numerous generators based on one or more criteria have been put forth.

Measurement For model selection and associated problems, biased distributions have been dynamically investigated in often occurring real experimental statistical data. Exponentiated Distributions techniques were applied to the Mukherjee Islam Distribution. There are several methods to represent a probability distribution, and the Mukherjee-Islam Distribution is defined by the conditional expectation of lower record values. The Mukherjee-Islam distribution was pioneered by Mukherjee and Islam (1983). These days, one of the most crucial aspects of reliability analysis is its finite range distribution. Because of its straightforward mathematical process and ease of management, it is preferred over more complicated distributions like the normal, Weibull beta, etc. The weighted Mukherjee-Islam distribution and its uses were covered by Dar et al. (2018). The various statistical features of the EMID were established by Rather and Subramanian (2018). Subramanian and Rather (2018) recently acquired weighted EMID. The production process can be observed using a variety of distributions; in this work, the EMID is utilized to monitor the process as shown in the control chart.

2. SIX SIGMA

Six Sigma is a set of techniques and tools used to improve business processes. It was introduced in 1986 by engineer Bill Smith while working at Motorola. Six Sigma practitioners use statistics, financial analysis, and project management to identify and reduce defects and errors, minimize variation, and increase quality and efficiency.

The Six Sigma method uses a step-by-step approach called DMAIC, an acronym that stands for Define, Measure, Analyze, Improve, and Control. According to Six Sigma adherents, a business may solve any seemingly unsolvable problem by following these five steps.

- ✓ *Define:* A team of people, led by a Six Sigma expert, chooses a process to focus on and defines the problem it wishes to solve.
- ✓ *Measure:* The team measures the initial performance of the process, creating a benchmark, and pinpoints a list of inputs that may be hindering performance.
- ✓ *Analyze:* Next the team analyzes the process by isolating each input, or potential reason for any failures, and testing it as the possible root of the problem.
- ✓ *Improve:* The team works from there to implement changes that will improve system performance.
- ✓ *Control:* The group adds controls to the process to ensure it does not regress and become ineffective once again.

Radhakrishnan and Sivakumaran (2008) used the concept of six sigma in the construction of sampling plans such as single, double and repetitive group sampling plans indexed through Six Sigma Quality Levels (SSQLs) with Poisson distribution as the base line distribution. Radhakrishnan (2009) suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on

Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan (2016) constructed control chart based on six sigma initiatives for Cumulative – Sum.

3. EXPONENTIATED MUKHERJEE-ISLAM DISTRIBUTION, OR EMID

The EMID PDF that Rather and Subramanian (2019) acquired is provided by

$f(x)=\frac{\alpha\beta x^{\alpha\beta^{-}}}{\theta^{\alpha\beta}}$, $0 < X < \theta, \alpha, \beta, \theta > 0$ (1)

The cumulative distribution function (cdf) of EMID is

$F(X) = \left(\left(\frac{X}{\theta}\right)^{\beta}\right)^{\alpha}$ (2)

4. THE DISTRIBUTION OF LENGTH BIASED EXPONENTIATED MUKHERJEE-ISLAM (LBEMI)

Let X be a real, arbitrary variable, and let f(x) be its probability density function (pdf). The probability density function of the weighted random variable X_w is given by if we assume that w(x) is the genuine weight function. $f_w(x) = \frac{w(x)f(x)}{E(w(x))}$, $x > 0$

Replace (1) We will obtain the necessary pdf of the LBEMI distribution in the following equation as

$f_1(x) = \frac{(\alpha\beta+1)x^{\alpha\beta}}{\theta^{\alpha\beta+}}$, $0 < x < \theta$ (3)

where the corresponding cdf of length-biased EMID is given as $E(x) = \alpha\beta\theta/(\alpha\beta+1)$

$F_1(x) = \int_0^x f_1(x)\, dx$ $F_1(x) = \left(\frac{X}{\theta}\right)^{\alpha\beta+1}$ (4)

Measures of the LBEMI Distribution's performance If X is the LBEMI distribution's random variable with parameters α, β, and θ, then the distribution's r-th order moment, E(X^r), may be obtained as

$E(X^r) = \mu_r^1 = \int_0^\theta x^r f_1(x)\, dx$

$E(X^r) = \int_0^\theta \frac{(\alpha\beta+1)x^{\alpha\beta+1}}{\theta^{\alpha\beta+1}}\, dx$

$E(X^r) = \int_0^\theta \frac{(\alpha\beta+1)\theta^{\alpha\beta+r+1}}{\theta^{\alpha\beta+1}(\alpha\beta+r+1)}\, dx$

$E(r) = \frac{(\alpha\beta+1)\theta^r}{(\alpha\beta+r+1)}$ (5)

Using r=1 in equation (5), we can get the mean of the LBEMI Distribution, which is given by

$E(x) = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)}$

Using r=2 in (5) we have

$E(x^2) = \frac{(\alpha\beta+1)\theta^2}{(\alpha\beta+3)}$

Following simplification, the LBEMI Distribution variance is

$V(x) = \frac{(\alpha\beta+1)\theta^2}{(\alpha\beta+3)(\alpha\beta+2)^2}$

5. LIMITATIONS OF CONTROL FOR THE LBEMI DISTRIBUTION

The control limits of LBEMI are as follows:

$$UCL = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)} + 3\frac{\theta}{(\alpha\beta+2)}\sqrt{\frac{(\alpha\beta+1)}{(\alpha\beta+3)}}$$

$$CL = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)}$$

$$LCL = \frac{(\alpha\beta+1)\theta}{(\alpha\beta+2)} - 3\frac{\theta}{(\alpha\beta+2)}\sqrt{\frac{(\alpha\beta+1)}{(\alpha\beta+3)}}$$

6. CONCLUSION

In this research article, the progression control has established the usage of EMID. Whilst the technique is non-normal distribution, a novel procedure is given for sentencing the method whilst production. The manipulation limits are given for the EMID with exceptional values of parameters α, β and θ. Generally, it is to built in the choice of the parameters based totally completely surely on the form of documents the manufacturing engineer is confronted with. The six-sigma based control chart is drawn by way of wondering about the generalized the parameters compared than the existing control three sigma-based control charts.

Acknowledgement

Nill

Funding

No funding was received to carry out this study.

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